

echoes is less prone to errors arising from change of pulse-echo shape with pressure than is that using gear which displays the rectified pulse-echoes. A typical data plot is given in Fig. 1 showing the difference between time of arrival of one of the maxima of echo No. 7 of the C_{44} wave in a 1.9-cm long copper crystal and a fixed time-marker, as a function of pressure gauge coil resistance. The pressure range indicated is about 9800 bars. Data points were taken about 5 min after each pressure change in order that the system be near thermal equilibrium. Absence of hysteresis justifies the time interval chosen. Strictly speaking one is interested only in the initial slopes of these curves, but experimentally all the plots are linear over the pressure range used. The slope of this line can be determined by a least squares method and this slope, together with the measured zero pressure transit time, enables one to compute the quantity $(nT_0)^{-1}dT_n/dR_g$, where T_0 is the zero pressure transit time, n is the number of the echo under observation, T_n is the observed time of arrival of the n th echo and R_g is the pressure gauge coil resistance. Since one can express the pressure gauge coil calibration as $dP = KdR_g$, and $T_n(P) = nT(P)$, the quantity $k^{-1}(nT_0)^{-1}dT_n/dR_g$ represents the fractional change of transit time with pressure, $(T_0)^{-1}dT/dP$.

A sequence of observations made with increasing pressure or with decreasing pressure will be called a run. Sets of runs made with a given transducer cemented in place will be called an experiment. Each value of $(T_0)^{-1}dT/dP$ which has been used is the result of at least two experiments each of which consisted of at least two runs. In the case of copper this procedure was followed for each of three crystals.

For crystals of nearly [110] orientation the equations relating transit times to elastic constants are given by

$$\begin{aligned} Y_2 &= B_s + 4\left(\frac{1}{3} - \Gamma\right)C' + 4\Gamma C, \\ Y_4 &= C + 2a_1(C' - C), \\ Y_5 &= C + 2a_2(C' - C), \end{aligned} \quad (1)$$

where

$$\begin{aligned} Y_2 &= \rho V_2^2 = 4\rho L^2/T_2^2, & Y_4 &= \rho V_4^2 = 4\rho L^2/T_4^2, \\ Y_5 &= \rho V_5^2 = 4\rho L^2/T_5^2, \end{aligned} \quad (2)$$

T_2 is the transit time for the longitudinal wave, and T_4 and T_5 refer to the slow and fast shear wave transit times, respectively. L is the length of the specimen between acoustic faces and ρ is the density of the material under study. The notation, $C = C_{44}$, $C' = (C_{11} - C_{12})/2$ and $B_s = (C_{11} + 2C_{12})/3$, has been used. B_s denotes the adiabatic bulk modulus. The quantities Γ , a_1 and a_2 are orientation functions¹⁴ which are independent of pressure for cubic materials. For orientations near [110], a_1 is about 0.5 and a_2 is nearly zero. For exactly [110] orientation one could write: $Y_4 = C'$, $Y_5 = C$. That is, C' is determined by Y_4 only, and C is found from Y_5 only. Taking the derivative with respect to pressure, of each equation relating the

Y 's and the elastic constants, one obtains

$$\begin{aligned} \frac{dY_2}{dP} &= \frac{dB_s}{dP} + 4\left(\frac{1}{3} - \Gamma\right)\frac{dC'}{dP} + 4\Gamma\frac{dC}{dP}, \\ \frac{dY_4}{dP} &= \frac{dC}{dP} + 2a_1\left(\frac{dC'}{dP} - \frac{dC}{dP}\right), \\ \frac{dY_5}{dP} &= \frac{dC}{dP} + 2a_2\left(\frac{dC'}{dP} - \frac{dC}{dP}\right). \end{aligned} \quad (3)$$

Taking the pressure derivative of the logarithm of any one of the equations relating the Y 's to the transit times, one obtains the relation

$$\frac{1}{Y} \frac{dY}{dP} = \frac{1}{\rho} \frac{d\rho}{dP} + \frac{2}{L} \frac{dL}{dP} - \frac{2}{T} \frac{dT}{dP}. \quad (4)$$

The first two terms on the right-hand side of the equation sum to $(3B_T)^{-1}$, where B_T is the isothermal bulk modulus, and the third term is the result of the measurements on changes of transit time with pressure, that is,

$$\frac{1}{Y} \frac{dY}{dP} = \frac{1}{3B_T} - \frac{2}{T} \frac{dT}{dP}, \quad (5)$$

with all quantities to be evaluated at zero pressure. Given the zero pressure values of all the Y 's and data on pressure variation of the transit times of the three waves, one can use the Eqs. (3) to compute the pressure derivatives of C , C' , and B_s at zero pressure. These equations determine the pressure derivatives of C and C' quite directly in the case of a [110] orientation, but the pressure derivative of B_s is derived from a combination of all three measurements.

The acoustic surface of each crystal was etched and a back reflection Laue x-ray taken after all acoustic measurements had been made. Ten spots were indexed and a least squares determination of the orientation was made.

The entire procedure outlined above was carried out for one crystal of each of silver and gold, and for two copper crystals of different lengths but similar orientation. Two copper crystals were used in order to form an estimate of the importance of any end effects such as nonhydrostatic stresses on the end of the specimen caused by differential compressibility of the quartz transducer and the metal, and possible change with pressure of acoustic end effects. The end effects proved to be less than the random experimental variations for copper, so that we felt reasonably safe in making measurements on one crystal only of each of silver and gold. These crystals just referred to were all within 2° of the [110] orientation. In addition, measurements were made on the pressure variation of the longitudinal wave transit time using a third copper crystal, near the [100] orientation.